VESIT.CZ
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# KAP 10/12 LOGARITMICKÉ ROVNICE

$$\log_a b = c$$

$$b = a^c$$

VZOREC

$$\log_4 x = \frac{3}{2}$$

$$x = 4^{\frac{3}{2}}$$

$$x = \sqrt{4^3}$$

$$x = \sqrt{64}$$

$$x = 8$$

$$K = \{8\}$$

$$\log(x-2) = \log(1)$$

$$\cancel{\log}(x-2) = \cancel{\log}(1)$$

$$x - 2 = 1 \qquad /+2$$

$$x = 3$$

$$K = \{3\}$$

Poznámka: Pokud se neuvede logaritmus, tak obecně platí, že je to logaritmus dekadický, tedy logaritmus při základu 10.

$$\boxed{\log(x)-\log(y)=\log\left(\frac{x}{y}\right)}$$ VZOREC

$$\log(0{,}5+x)=\log 0{,}5-\log x$$

$$\cancel{\log}(0{,}5+x)=\cancel{\log}\frac{0{,}5}{x}$$

$$0{,}5+x=\frac{0{,}5}{x} \qquad /\cdot x$$

$$0{,}5x+x^2=0{,}5 \qquad /-0{,}5$$

$$x^2+0{,}5x-0{,}5=0 \qquad /\cdot 2$$

$$2x^2+x-1=0$$

$$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x_{1,2}=\frac{-1\pm\sqrt{1^2-4\cdot 2\cdot(-1)}}{2\cdot 2}$$

$$x_{1,2}=\frac{-1\pm\sqrt{1-(-8)}}{4}$$

$$x_{1,2}=\frac{-1\pm 3}{4} \quad \begin{cases} x_1=\frac{1}{2} \\ x_2=-1 \end{cases}$$

$$\left(x-\frac{1}{2}\right)\left(x-(-1)\right)$$

$$x_0=\frac{1}{2} \qquad x_0=-1$$

$$\underline{\underline{K=\{0{,}5\}}}$$

pro logaritmus platí:

$$Df=R^+$$

! VÝSLEDKEM NEMŮŽE BÝT −1 PROTOŽE VIZ ZADÁNÍ $\log(0{,}5+(-1)) \Rightarrow$

$\Rightarrow \log(-0{,}5) \Rightarrow$

$\Rightarrow$ ERROR

LOGARITMUS NESMÍ BÝT ZÁPORNÝ ANI NULA

$$\underline{\log(7x+6) = 1 + \log(3x-4)}$$

$$\log(7x+6) = \log_{10}10 + \log(3x-4)$$

$$\cancel{\log}(7x+6) = \cancel{\log}(30x-40)$$

$$7x+6 = 30x-40 \qquad /-30x$$

$$-23x+6 = -40 \qquad /-6$$

$$-23x = -46 \qquad /\cdot(-1)$$

$$23x = 46 \qquad /:23$$

$$x = 2$$

$$\underline{\underline{K = \{2\}}}$$

VZORCE:

$$\boxed{\log(x) + \log(y) = \log(x \cdot y)}$$

$$\boxed{\log_a a = 1}$$

$$\boxed{1 = \log_a a}$$

$$\log_2 x - \log_2 \sqrt{x} + \log_2 \frac{1}{x} = -1$$

$$\log_2 x - \log_2 \sqrt{x} + \log_2 \frac{1}{x} = -\log_2 2$$

$$\log_2 x - \log_2 x^{\frac{1}{2}} + \log_2 \frac{1}{x} = -\log_2 2$$

$$\log_2 x - \frac{1}{2}\log_2 x + \log_2 \frac{1}{x} = -\log_2 2$$

$$\log_2 x - \frac{1}{2}\log_2 x + \log_2 \frac{1}{x} = -\log_2 2$$

$$\log_2 \left(x \cdot \frac{1}{x}\right) - \frac{1}{2}\log_2 x = -\log_2 2$$

$$\log_2 (1) - \frac{1}{2}\log_2 x = -\log_2 2$$

$$0 - \frac{1}{2}\log_2 x = -\log_2 2 \quad / \cdot (-2)$$

$$\log_2 x = 2\log_2 2$$

$$\log_2 x = \log_2 2^2$$

$$\cancel{\log_2} x = \cancel{\log_2} 4$$

$$x = 4$$

$$K = \{4\}$$

$$\boxed{\log a^b = b \cdot \log a}$$

$$\boxed{b \cdot \log a = \log a^b}$$

$$\boxed{\log_a 1 = 0}$$

Poznámka: LOGARITMUS PŘI ZÁKLADĚ DVOU

$$\log_2 x = 2\log_2 2$$

$$\log_2 x = 2 \cdot 1$$

$$\log_2 x = 2$$

$$2^2 = x$$

$$4 = x$$

# LOGARITMICKÉ ROVNICE I SE ZADANOU URČITOU PODMÍNKOU

$$a = \log(b+2) - 1$$
$$a = 0$$

ZADÁNÍ

$$0 = \log(b+2) - 1 \qquad /+1$$

$$1 = \log(b+2)$$

$$\cancel{\log_{10}} 10 = \cancel{\log}(b+2)$$

$$10 = b + 2 \qquad /-2$$

$$8 = b$$

$$K = \{8\}$$

Jestli a je rovno nule, tak b je rovno osmi.

$$a = \log(b+2) - 1$$
$$b = 0$$

$$a = \log(0+2) - 1$$

$$a = \log(2) - 1$$

$$a = \log 2 - \log_{10} 10$$

$$a = \log\left(\frac{2}{10}\right)$$

$$a = \log\frac{1}{5}$$

$$K = \left\{\log\frac{1}{5}\right\}$$

$$\log(3x-6)-\log 5=0$$

$$\log(3x-6)-\log 5=0 \quad /+\log 5$$

$$\cancel{\log}(3x-6)=\cancel{\log}5$$

$$3x-6=5 \quad /+6$$

$$3x=11 \quad /:3$$

$$x=\frac{11}{3}$$

$$\underline{\underline{K=\left\{\frac{11}{3}\right\}}}$$

ŘEŠTE LOGARITMICKÉ ROVNICE v R

a) $$\frac{\log(2x+10)}{2} = \log(x+1)$$

b) $$\log 2x - \log \sqrt[3]{x} + \log x^2 = \log 2 - \log x^{\frac{1}{3}} + 1$$

c) $$\log_4(x+3) - \log_4(x-1) = 2 - \log_4 8$$

a) $\frac{\log(2x+10)}{2} = \log(x+1)$

$\frac{\log(2x+10)}{2} = \log(x+1) \quad /\cdot 2$

$\log(2x+10) = 2\log(x+1)$

$\log(2x+10) = \log(x+1)^2$

$2x+10 = (x+1)^2$

$2x+10 = x^2+2x+1 \quad /-2x$

$10 = x^2+1 \quad /-1$

$9 = x^2 \quad /-x^2$

$-x^2+9 = 0 \quad /\cdot(-1)$

$x^2-9 = 0$

$(x+3)\ (x-3)$

$x_0 = -3 \quad x_0 = 3$

$$\underline{\underline{K = \{3\}}}$$

$\log(-3+1) \Rightarrow \log(-2)$
$\Downarrow$
ERROR

PRO LOGARITMUS
PLATÍ $D_f = R^+$

b)

$$\log 2x - \log \sqrt[3]{x} + \log x^2 = \log 2 - \log \frac{1}{x^3} + 1$$

$$\log 2x - \log x^{\frac{1}{3}} + 2\log x = \log 2 - \log x^{-3} + \log_{10} 10$$

$$\log 2x - \frac{1}{3}\log x + 2\log x = \log 2 - (-3)\log x + \log_{10} 10$$

$$\log 2x - \frac{1}{3}\log x + 2\log x = \log 2 + 3\log x + \log_{10} 10 \quad / -3\log x$$

$$-3\log x + \log 2x - \frac{1}{3}\log x + 2\log x = \log 2 + \log_{10} 10 \quad / -\log 2x$$

$$-3\log x - \frac{1}{3}\log x + 2\log x = \log 2 - \log 2x + \log_{10} 10$$

Cíl: dostat hodnoty před logaritmem na jednu stranu, hodnoty před logaritmem lze sčítat.

$$\left(-\frac{3}{1} - \frac{1}{3} + \frac{2}{1} = \frac{-9-1+6}{3} = -\frac{4}{3}\right)$$

$$-\frac{4}{3}\log x = \log 2 - \log 2x + \log_{10} 10$$

$$-\frac{4}{3}\log x = \log\left(\frac{2}{2x}\right) + \log_{10} 10$$

$$-\frac{4}{3}\log x = \log\left(\frac{1}{x}\right) + \log_{10} 10$$

$$-\frac{4}{3}\log x = \log x^{-1} + \log_{10} 10$$

$$-\frac{4}{3}\log x = -1\log x + \log 10$$

$$-\frac{4}{3}\log x = -\log x + \log 10 \quad / +\log x$$

$$-\frac{4}{3}\log x + \log x = \log 10$$

$$-\frac{1}{3}\log x = \log 10$$

$$\cancel{\log} x^{-\frac{1}{3}} = \cancel{\log} 10$$

$$x^{-\frac{1}{3}} = 10$$

$$\frac{1}{x^{\frac{1}{3}}} = 10 \qquad / \cdot x^{\frac{1}{3}}$$

$$1 = 10x^{\frac{1}{3}} \qquad /:10$$

$$\frac{1}{10} = x^{\frac{1}{3}}$$

$$x^{\frac{1}{3}} = \frac{1}{10}$$

$$\sqrt[3]{x} = \frac{1}{10}$$

/ PŘIDÁM MOCNINU NA TŘETÍ NA OBĚ STRANY

$$\left(\sqrt[3]{x}\right)^3 = \left(\frac{1}{10}\right)^3$$

$$x = \left(\frac{1}{10}\right)^3$$

$$x = \frac{1^3}{10^3}$$

$$x = \frac{1}{1000}$$

$$K = \left\{\frac{1}{1000}\right\}$$

c)

$$\log_4(x+3) - \log_4(x-1) = 2 - \log_4 8$$

$$\log_4(x+3) - \log_4(x-1) = 2\log_4 4 - \log_4 8$$

$$\log_4\left(\frac{x+3}{x-1}\right) = \log_4 4^2 - \log_4 8$$

$$\log_4 1 + \frac{4}{x-1} = \log_4 \frac{4^2}{8}$$

$$\log_4 1 + \frac{4}{x-1} = \log_4 \frac{16}{8}$$

$$\cancel{\log_4} 1 + \frac{4}{x-1} = \cancel{\log_4} 2$$

$$1 + \frac{4}{x-1} = 2$$

$$\frac{1}{1} + \frac{4}{x-1} = 2$$

$$\frac{x-1+4}{x-1} = 2$$

$$\frac{x+3}{x-1} = 2 \qquad /\cdot(x-1)$$

$$x+3 = 2(x-1)$$

$$x+3 = 2x-2 \qquad /-2x$$

$$-x+3 = -2 \qquad /-3$$

$$-x = -5 \qquad /\cdot(-1)$$

$$x = 5$$

Poznámka:

$$(x+3):(x-1) = 1 + \frac{4}{x-1}$$

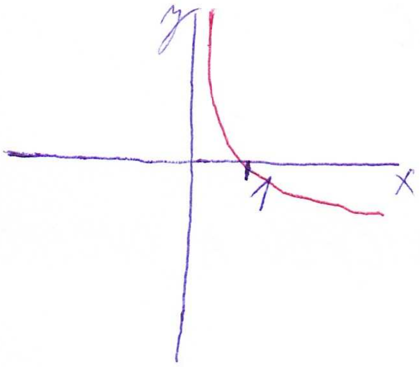
$$\frac{-(x-1)}{4}$$

$$\boxed{K = \{5\}}$$

# LOGARITMICKÁ FCE

PŘÍKLAD: $g: y = 4\log_{\frac{1}{4}}(x-1) - 2$

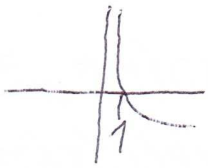
① $\log_{\frac{1}{4}}$



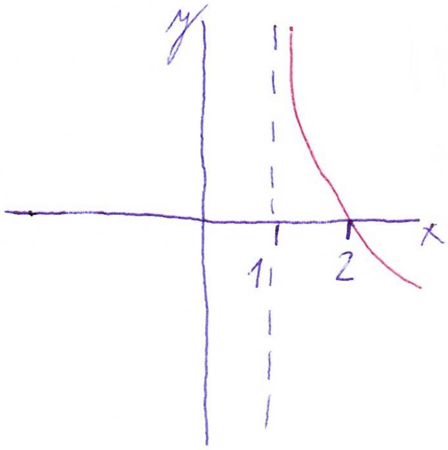

$$\boxed{y = \log_d x}$$


$d > 1$ ROSTOUCÍ

$0 < d < 1$ KLESAJÍCÍ




② $\log_{\frac{1}{4}} \overbrace{(x-1)}^{\text{argument}}$

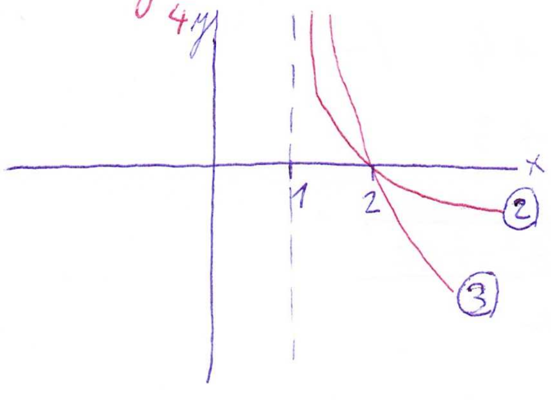



$$\boxed{y = \log(x+c)}$$

$c > 0$ C JE KLADNÉ ←

$c < 0$ C JE ZÁPORNÉ →

③ $4\log_{\frac{1}{4}}(x-1)$



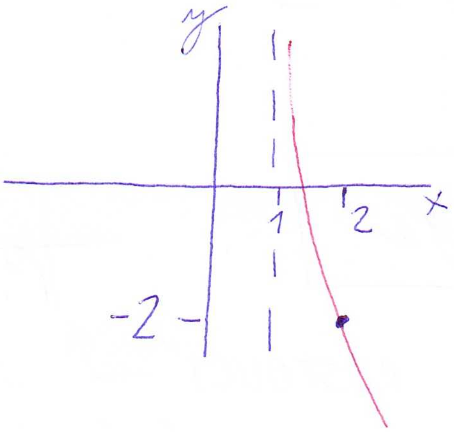

Z DŮVODU, ŽE 4 KRÁT JE VĚTŠÍ, GRAF SE NÁM JAKOBY NAROVNÁ, BUDE STRMĚJŠÍ ③

4 $4\log_{\frac{1}{4}}(x-1)-2$

$$\boxed{y=\log(x)+b}$$

$b>0$ ↑ B JE KLADNÉ

$b<0$ ↓ B JE ZÁPORNÉ




GRAF POSUNEO 2 DOLŮ

VIDÍME, ŽE JE PRŮSEČÍK MEZI JEDNIČKOU A DVOJKOU, VYPOČÍTÁM HO.

$P_x = ?$ $\quad y=0$

$$y = 4\log_{\frac{1}{4}}(x-1)-2$$

$$0 = 4\log_{\frac{1}{4}}(x-1)-2 \quad /+2$$

$$2 = 4\log_{\frac{1}{4}}(x-1)$$

$$2\log_{\frac{1}{4}}\frac{1}{4} = 4\log_{\frac{1}{4}}(x-1)$$

$$\cancel{\log_{\frac{1}{4}}}\left(\frac{1}{4}\right)^2 = \cancel{\log_{\frac{1}{4}}}(x-1)^4$$

$$\left(\frac{1}{4}\right)^{\cancel{2}} = (x-1)^{\cancel{4}}$$

$$\frac{1}{4} = (x-1)^2$$

$$\frac{1}{4} = x^2-2x+1 \quad /\cdot 4$$

$$1 = 4x^2-8x+4 \quad /-1$$

$$0 = 4x^2-8x+3$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2\cdot a}$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{1,2} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 4 \cdot 3}}{2 \cdot 4}$$

$$x_{1,2} = \frac{8 \pm \sqrt{64 - 48}}{8}$$

$$x_{1,2} = \frac{8 \pm 4}{8} \quad x_1 = \frac{12}{8} = \frac{3}{2} \quad x_2 = \frac{4}{8} = \frac{2}{4} = \frac{1}{2}$$

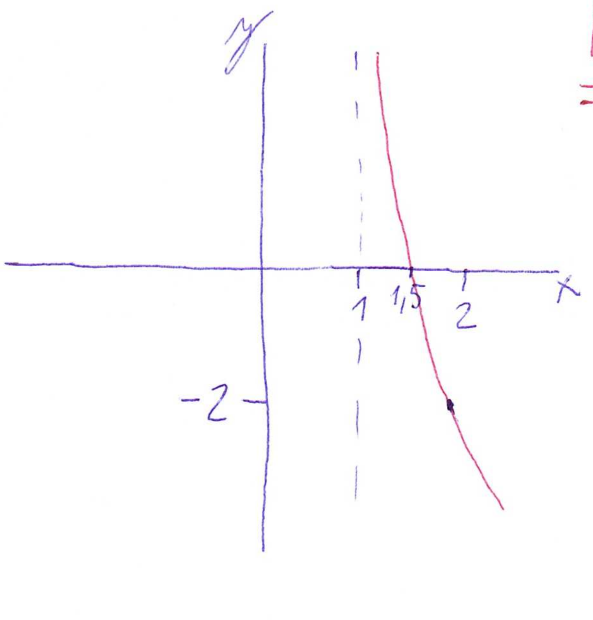
$$\left(x - \frac{3}{2}\right)\left(x - \frac{1}{2}\right)$$

$$x_0 = \frac{3}{2} \qquad x_0 = \frac{1}{2}$$

Při dosazení do zadání, nesmí vycházet záporné číslo.

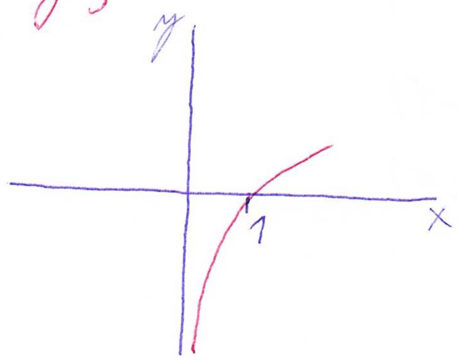
$$K = \left\{\frac{3}{2}\right\}$$

$$\underline{\underline{P_x = \left[\frac{3}{2} ; 0\right]}}$$

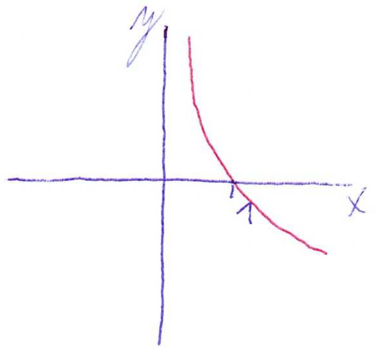


PŘÍKLAD: $f: y = -\log_3(x-1) + 2$

1) $\log_3 x$




2) $-\log_3 x$

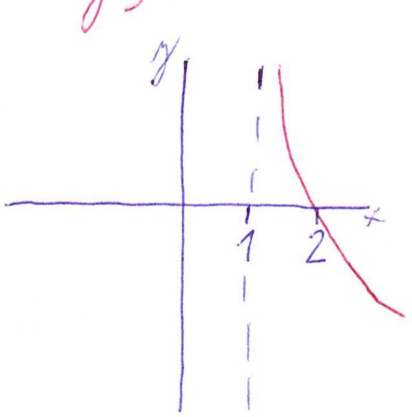



$$\boxed{y = a \cdot \log(x)}$$

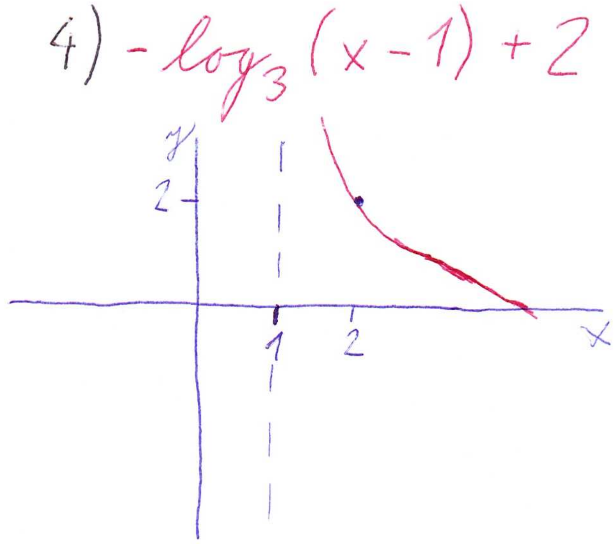
$\pm a \Rightarrow$ PŘEKLOPENÍ

$a \Rightarrow$ ODDÁLENÍ NEBO PŘIBLÍŽENÍ K OSÁM, ČÍM VĚTŠÍ $a$, TÍM SE ODDALUJE OD OS.

3) $-\log_3 \cancel{x} (x-1)$




4) $-\log_3(x-1) + 2$



$P_x = ?$ $\quad y = 0$

$$0 = -\log_3(x-1) + 2 \quad /-2$$

$$-2 = -\log_3(x-1)$$

$$-2\log_3 3 = -\log_3(x-1)$$

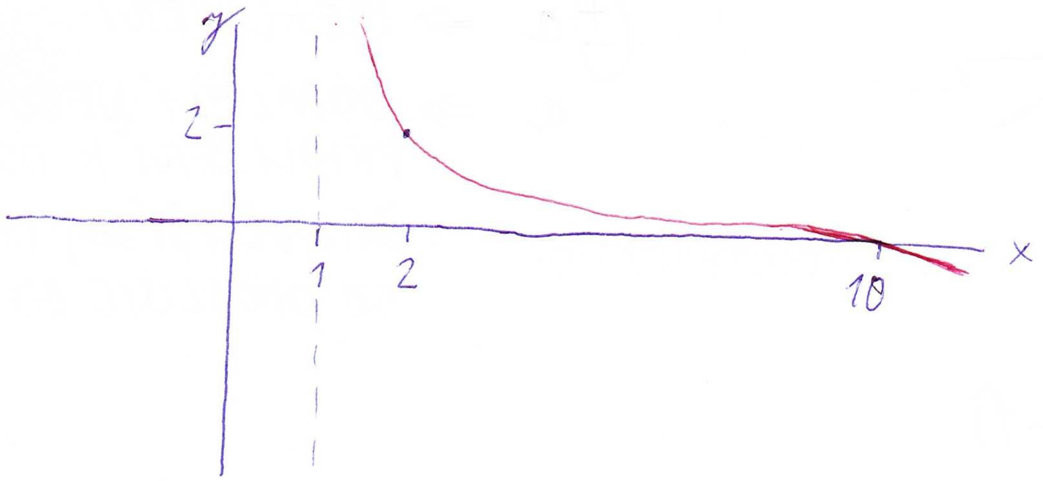
$$\cancel{-\log_3} 3^2 = \cancel{-\log_3}(x-1)$$

$$3^2 = x - 1$$

$$9 = x - 1 \quad /+1$$

$$10 = x$$

$$\underline{\underline{P_x = [10; 0]}}$$



# LOGARITMICKÉ FCE

Logaritmus má vždy průsečík s osou x.
Logaritmická fce je inverzní k exponenciální funkci a naopak.

$$y = \ln(x) = \log_e(x)$$

EULEROVO ČÍSLO ≐ 2,72

$$e^y = x$$

PŘ: $e^y = 5$ $\Rightarrow \ln 5 \Rightarrow \doteq 1{,}61 \Rightarrow e^{1{,}61} \doteq 5$

$a^c = b$ $\Rightarrow \log_a b = c$

$\log_e 5 = y$

! $\log 0 \Rightarrow$ neexistuje

$$\log_a b = c$$

ARGUMENT

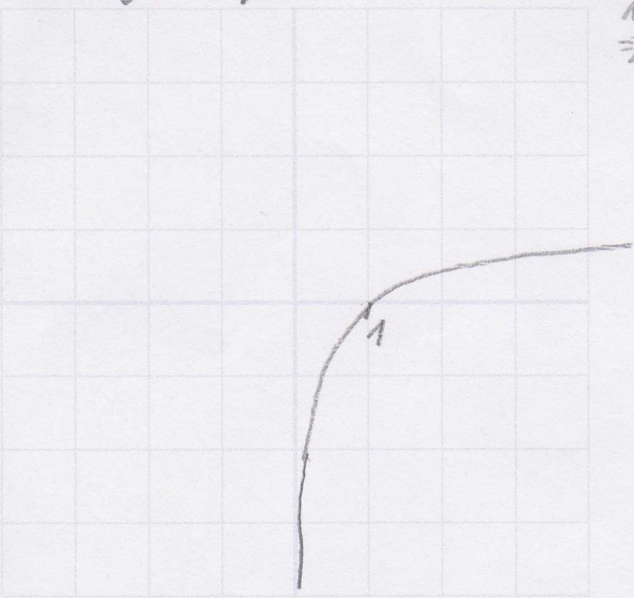
ZÁKLAD

LOGARITMUS

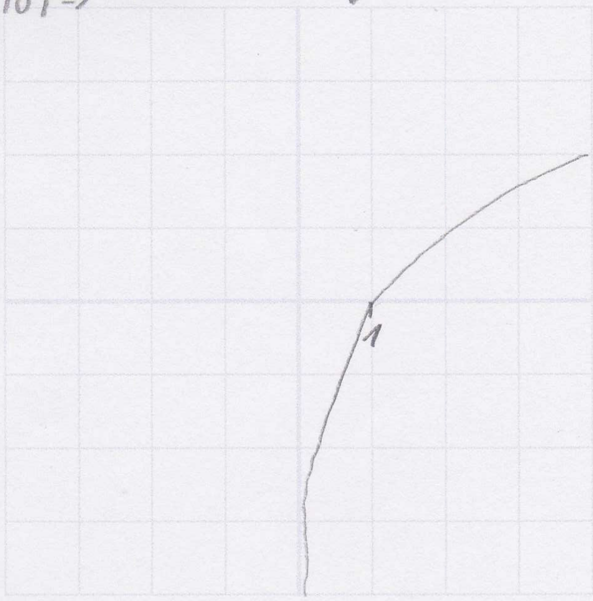
4a

$y = \log x$ $(\ln x)$

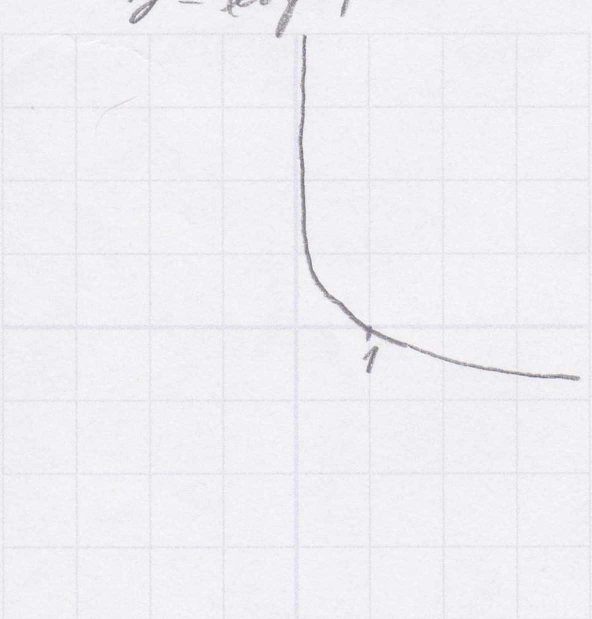
logaritmus log přetočí v $x=10$, $\Rightarrow$ $\Rightarrow y>1$



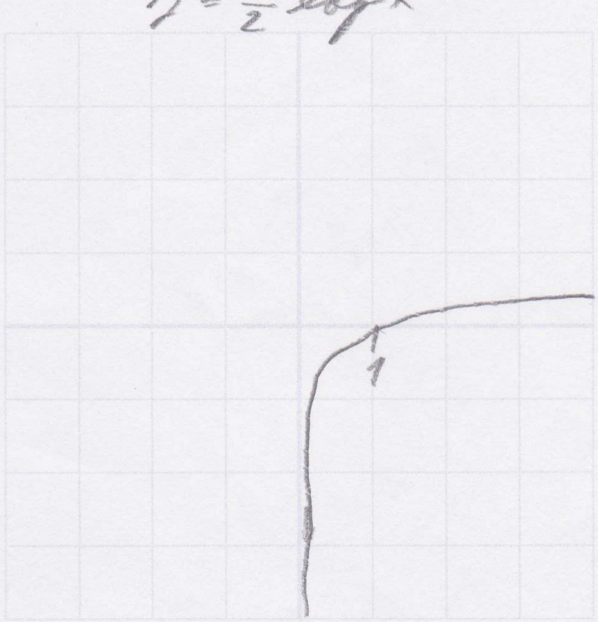

$y = 3 \log x$



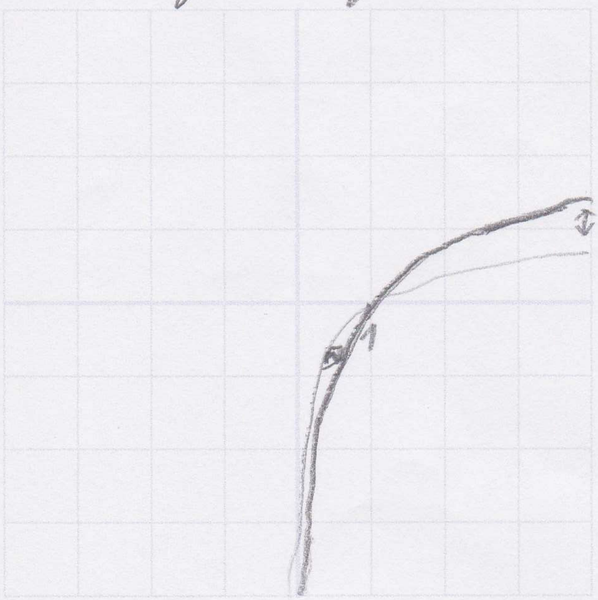

$y = -(\log x)$



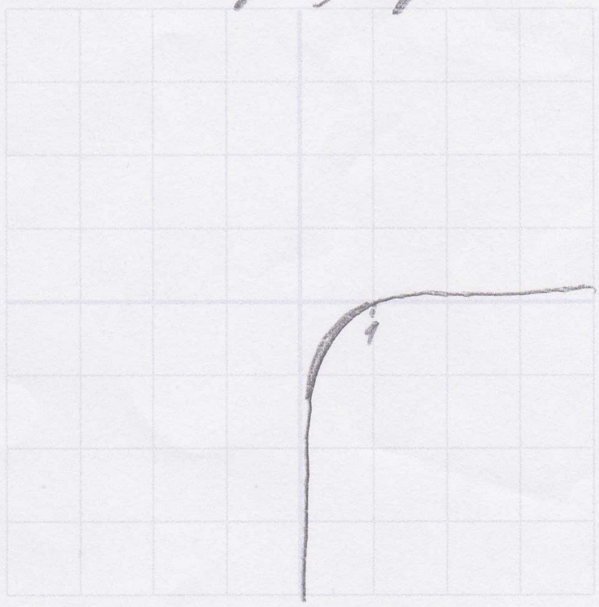

$y = \frac{1}{2} \log x$




$y = 2 \cdot \log(x)$ deformace

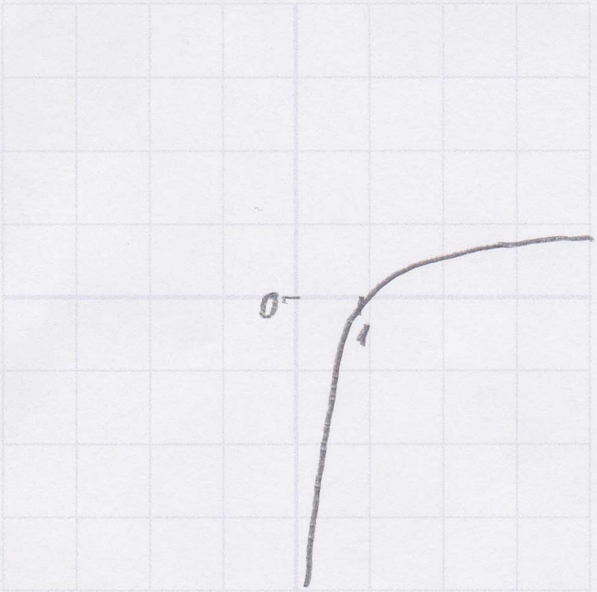



$y = \frac{1}{3} \log x$

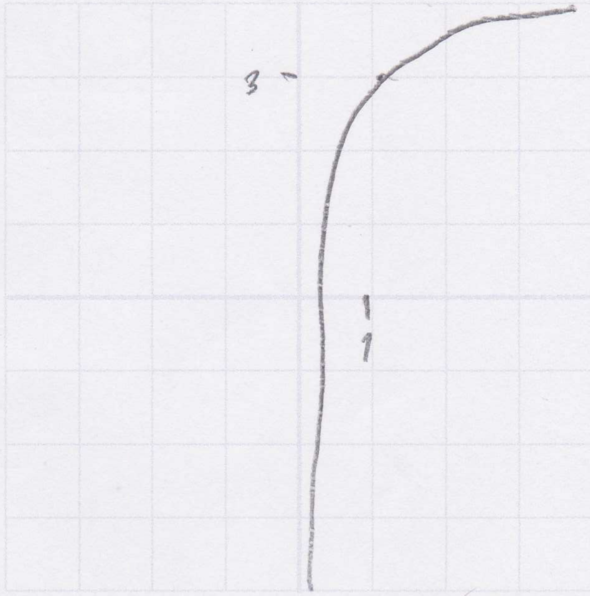


4b

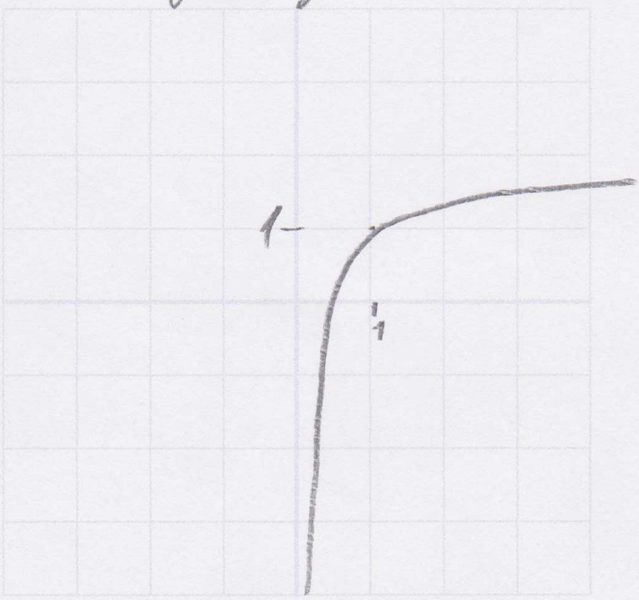
$y = \log x$



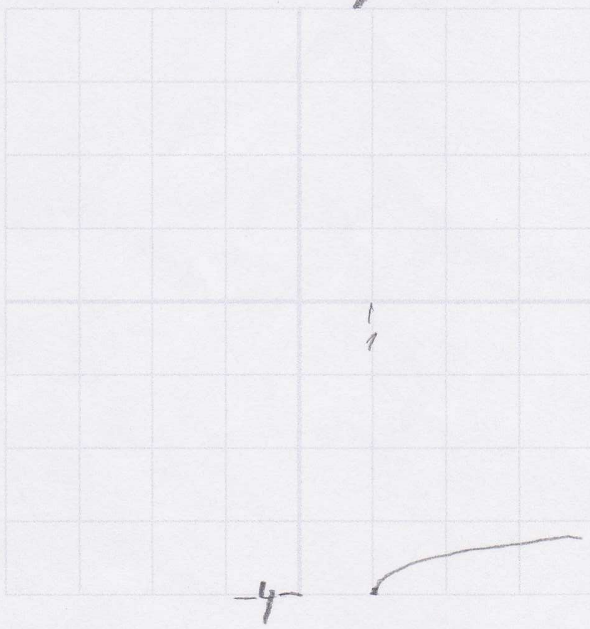

$y = \log x + 3$



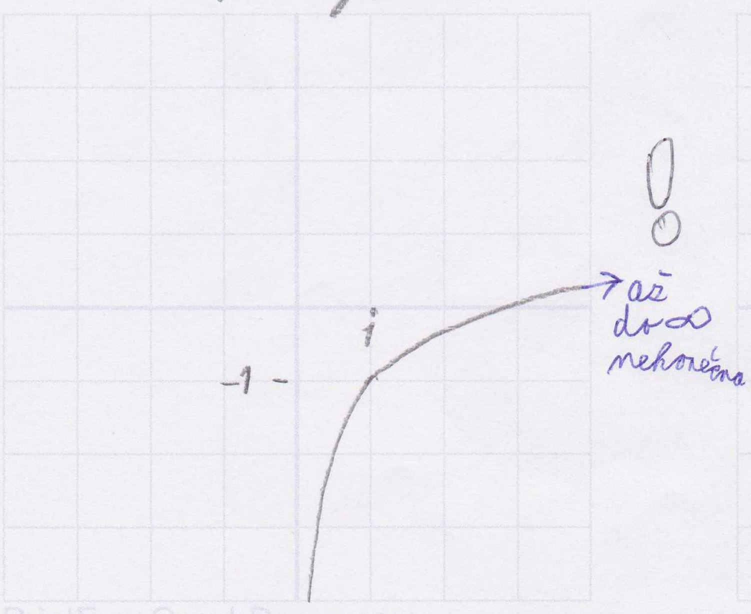

$y = \log x + 1$




$y = \log x - 4$

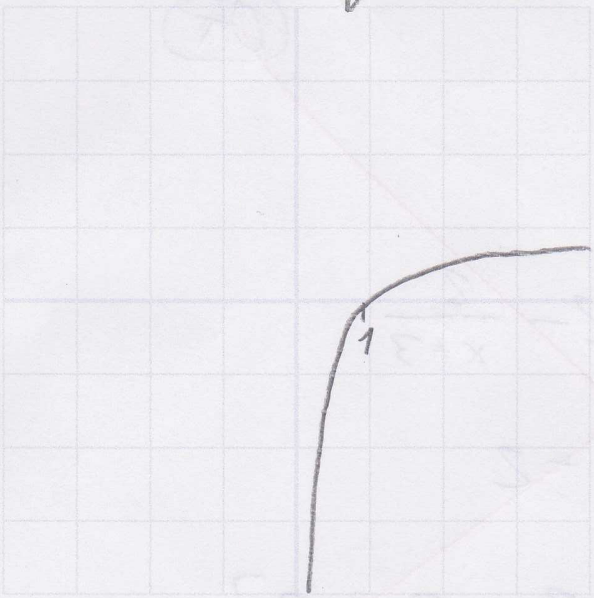



$y = \log x - 1$

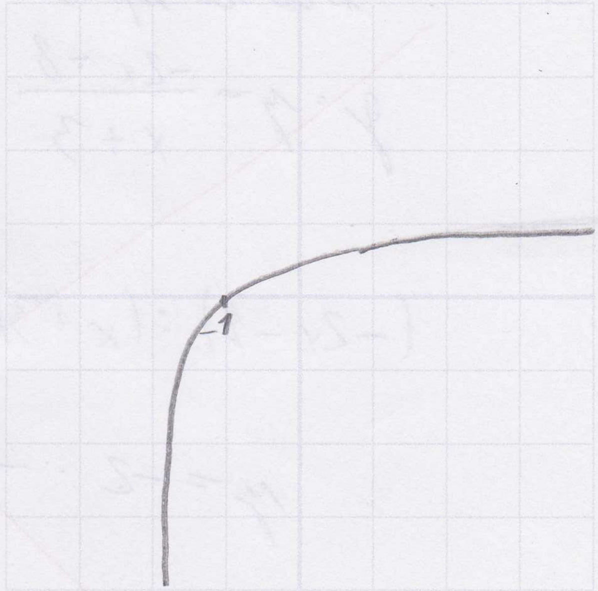


4c

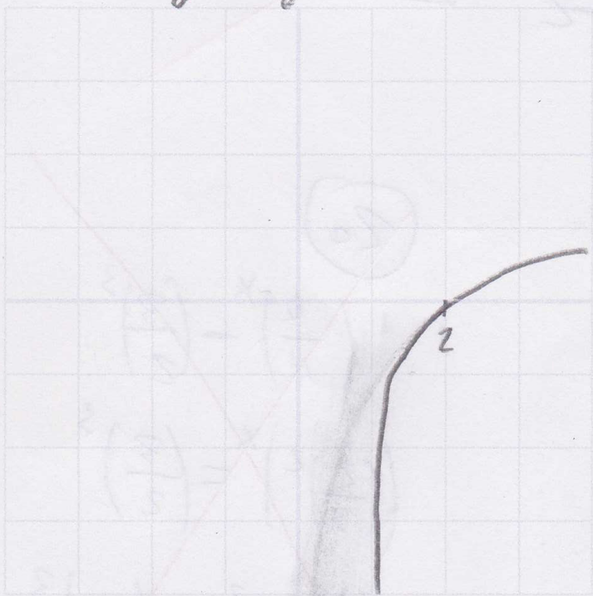
$y = \log x$



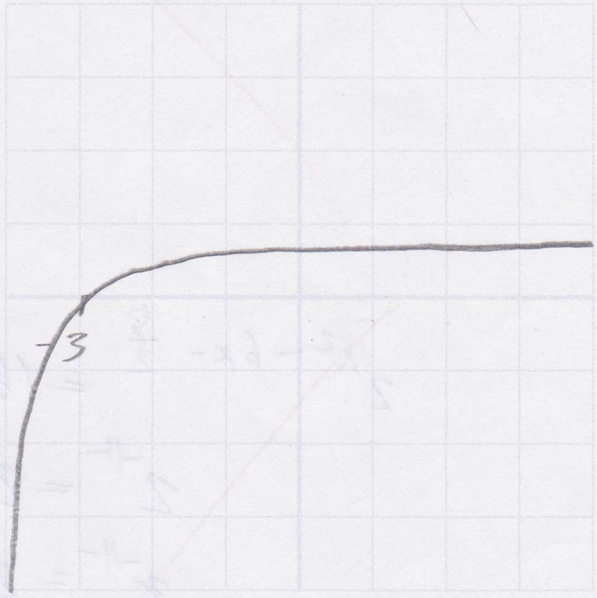

$y = \log(x+2)$




$y = \log(x-1)$




$y = \log(x+4)$




$y = \log(x-3)$