SKALÁRNÍ SOUČIN vektorů

$\boxed{\cdot}$

$\vec{u}(u_1\, u_2\, u_3)$ $\vec{v}(v_1\, v_2\, v_3)$

$$\vec{u}\cdot\vec{v} = u_1\cdot v_1 + u_2\cdot v_2 + u_3\cdot v_3$$

PŘ: $u = (3)$ $v = (100)$

$$u\cdot v = 3\cdot 100 = \underline{\underline{300}}$$

PŘ: $u = (1, -1, 2, 3)$ $(1,-1,2,3)\cdot(1,1,-3,0) = ?$

$v = (1, 1, -3, 0)$

$$u\cdot v = 1\cdot 1 + (-1)\cdot 1 + 2\cdot(-3) + 3\cdot 0 =$$
$$= 1 - 1 - 6 = \underline{\underline{-6}}$$

$$\begin{pmatrix}1 & -1 & 2 & 3\end{pmatrix}\cdot\begin{pmatrix}1\\1\\-3\\0\end{pmatrix} = \underline{\underline{-6}}$$

VEKTOROVÝ SOUČIN VEKTORŮ

$\boxed{\times}$

$\vec{u} = (u_1, u_2, u_3)$

$\vec{v} = (v_1, v_2, v_3)$

$\vec{w} = \vec{u} \times \vec{v} = (u_2 \cdot v_3 - u_3 \cdot v_2,\ u_3 \cdot v_1 - u_1 \cdot v_3,\ u_1 v_2 - u_2 v_1)$

PŘ:

$\vec{u} = (3, 1, 2)$

$\vec{v} = (2, 3, 1)$

$1 \cdot 1 - 2 \cdot 3 = -5$

$2 \cdot 2 - 3 \cdot 1 = 1$

$3 \cdot 3 - 1 \cdot 2 = 7$

$\underline{\underline{\vec{w} = (-5, 1, 7)}}$

PŘ:

$\vec{u} = (2, 3, 5)$

$\vec{v} = (1, 2, 2)$

$(2, 3, 5) \times (1, 2, 2) = ?$

$3 \cdot 2 - 5 \cdot 2 = -4$

$5 \cdot 1 - 2 \cdot 2 = 1$

$2 \cdot 2 - 3 \cdot 1 = 1$

$\underline{\underline{\vec{w} = (-4, 1, 1)}}$

PŘ: $\vec{u} = (1, 1, 0)$

$\vec{v} = (-3, 0, 1)$

$1 \cdot 1 - 0 \cdot 0 = 1$

$0 \cdot (-3) - 1 \cdot 1 = -1$

$1 \cdot 0 - 1 \cdot (-3) = 3$

$\underline{\underline{\vec{w} = (1, -1, 3)}}$

VEKTORY JSOU LINEÁRNĚ ZÁVISLÉ NEBO LINEÁRNĚ NEZÁVISLÉ?

$\vec{u}_1 = (1, -1, 0, 1, 2)$ $\vec{u}_2 = (0, 2, -1, 1, 0)$ $\vec{u}_3 = (0, 0, 1, 0, -1)$

$$\begin{pmatrix} 1 & 0 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 1 \\ 1 & 1 & 0 \\ 2 & 0 & -1 \end{pmatrix}$$

$$\alpha_1 \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 2 \end{pmatrix} + \alpha_2 \begin{pmatrix} 0 \\ 2 \\ -1 \\ 1 \\ 0 \end{pmatrix} + \alpha_3 \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}$$

$$\begin{aligned} \alpha_1 \qquad\qquad &= 0 \\ -\alpha_1 + 2\alpha_2 \qquad &= 0 \\ -\alpha_2 + \alpha_3 &= 0 \\ \alpha_1 + \alpha_2 \qquad &= 0 \\ 2\alpha_1 \qquad - \alpha_3 &= 0 \end{aligned}$$

$\Longrightarrow$ 1. KROK $\underline{\alpha_1 = 0}$

$\Rightarrow$ JAKO 2. KROK $-\alpha_1 + 2\alpha_2 = 0$

$-0 + 2\alpha_2 = 0$

$2\alpha_2 = 0 \; /:2$

$\underline{\alpha_2 = 0}$

VYBERU JAKO 3. KROK, ABYCH ZJISTIL α_3:

$-\alpha_2 + \alpha_3 = 0$

$-0 + \alpha_3 = 0$

$\underline{\alpha_3 = 0}$

PO DOSAZENÍ BYCH DOSTAL:

$0 = 0$

$0 = 0$

$0 = 0$

$0 = 0$

$0 = 0$

JEDINÉ ŘEŠENÍ JE NULOVÉ, PROTO VEKTORY JSOU LINEÁRNĚ NEZÁVISLÉ (LNZ)

JINÝ POSTUP: VEKTORY JSOU LZ NEBO LNZ?
SE STEJNÝM
ZADÁNÍM

$\vec{u}_1 = (1, -1, 0, 1, 2) \quad \vec{u}_2 = (0, 2, -1, 1, 0) \quad \vec{u}_3 = (0, 0, 1, 0, -1)$

$$\begin{pmatrix} 1 & 0 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 1 \\ 1 & 1 & 0 \\ 2 & 0 & -1 \end{pmatrix} = x \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 2 \end{pmatrix} + y \begin{pmatrix} 0 \\ 2 \\ -1 \\ 1 \\ 0 \end{pmatrix} + z \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 2 & 0 & -1 & 0 \end{array}\right) \begin{array}{l} \\ \cdot 2 \\ \\ \\ + \end{array} \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 4 & -1 & 0 \end{array}\right) \begin{array}{l} \\ \cdot 1 \\ \\ + \\ \\ \end{array} \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 4 & -1 & 0 \end{array}\right) \begin{array}{l} \\ \\ \\ \cdot \frac{1}{3} \\ \\ \end{array} \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 4 & -1 & 0 \end{array}\right) \begin{array}{l} \\ \\ \\ \cdot(-4) \\ + \end{array} \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right) \begin{array}{l} \\ + \\ \\ \cdot(-2) \\ \\ \end{array} \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right) \begin{array}{l} \\ \\ \\ + \\ \cdot 1 \end{array} \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right) \begin{array}{l} + \\ \cdot 1 \\ \\ \\ \\ \end{array} \sim \left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right) \begin{array}{l} \\ \\ \cdot 1 \\ + \\ \\ \end{array} \sim \left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right) \begin{array}{l} \\ \cdot(-1) \\ \cdot(-1) \\ \\ \cdot(-1) \end{array} \sim$$

$$\sim \left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

$x = 0$
$y = 0$
$z = 0$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

VÝSLEDKEM JE
NULOVÝ VEKTOR

LNZ

Vektory jsou LZ nebo LNZ?

$\vec{x} = (1, 2, 0)$ $\quad \vec{y} = (-1, -2, 1)$ $\quad \vec{z} = (1, 1, 1)$

$$\begin{pmatrix} 1 & -1 & 1 \\ 2 & -2 & 1 \\ 0 & 1 & 1 \end{pmatrix} = x \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + y \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix} + z \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{aligned} x - y + z &= 0 \\ 2x - 2y + z &= 0 \\ y + z &= 0 \end{aligned}$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 2 & -2 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right) \cdot(-2) \sim \left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right) \cdot(-1) \sim \left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

$$\begin{aligned} x - y + z &= 0 \quad \Rightarrow \quad x - 0 + 0 = 0, \ \underline{x = 0} \\ y + z &= 0 \quad \Rightarrow \quad y + 0 = 0, \ \underline{y = 0} \\ \underline{z} &\underline{= 0} \end{aligned}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} *$$

LNZ = výsledkem je NULOVÝ VEKTOR*

Při dosazování vyjdou jen nuly $\Rightarrow$

$\Rightarrow$ LNZ

VEKTORY JSOU LINEÁRNĚ ZÁVISLÉ NEBO LINEÁRNĚ NEZÁVISLÉ?

$\vec{u} = (3; -1; 0) \quad \vec{v} = (-2; 2; 1) \quad \vec{w} = (0; 4; 3)$

$$\begin{pmatrix} 3 & -2 & 0 \\ -1 & 2 & 4 \\ 0 & 1 & 3 \end{pmatrix} = x\begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} + y\begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} + z\begin{pmatrix} 0 \\ 4 \\ 3 \end{pmatrix}$$

$$\begin{aligned} 3x - 2y \quad &= 0 \\ -x + 2y + 4z &= 0 \\ y + 3z &= 0 \end{aligned}$$

$$\left(\begin{array}{ccc|c} 3 & -2 & 0 & 0 \\ -1 & 2 & 4 & 0 \\ 0 & 1 & 3 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} -1 & 2 & 4 & 0 \\ 3 & -2 & 0 & 0 \\ 0 & 1 & 3 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} -1 & 2 & 4 & 0 \\ 0 & 1 & 3 & 0 \\ 3 & -2 & 0 & 0 \end{array}\right) \begin{array}{l} \cdot 3 \\ \\ + \end{array} \sim$$

$$\sim \left(\begin{array}{ccc|c} -1 & 2 & 4 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 4 & 12 & 0 \end{array}\right) \begin{array}{l} \\ \cdot(-4) \\ + \end{array} \sim \left(\begin{array}{ccc|c} -1 & 2 & 4 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \cdot(-1) \sim \left(\begin{array}{ccc|c} 1 & -2 & -4 & 0 \\ 0 & 1 & 3 & 0 \\ \boxed{0} & \boxed{0} & \boxed{0} & \boxed{0} \end{array}\right)$$

NEKONEČNĚ MNOHO ŘEŠENÍ

$$\begin{aligned} x - 2y - 4z &= 0 \\ y + 3z &= 0 \end{aligned}$$

ZVOLÍM SI ZA z PARAMETR

$\underline{z = s} \quad s \in R$

$y + 3s = 0$

$\underline{y = -3s}$

$x - 2(-3s) - 4s = 0$

$x + 6s - 4s = 0$

$x + 2s = 0 \quad / -2s$

$\underline{x = -2s}$

VÝSLEDEK

$$\boxed{\begin{aligned} x &= -2s \quad \text{LZ} \\ y &= -3s \\ z &= s \quad s \in R \end{aligned}}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2s \\ -3s \\ s \end{pmatrix} = s \cdot \begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix}^{*}$$

VÝSLEDKEM NENÍ NULOVÝ VEKTOR*, PROTO JSOU LINEÁRNĚ ZÁVISLÉ

Řešením může být i například:

$$x = 2 \quad y = 3 \quad z = -1$$

$$s \cdot \begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix} = -1 \cdot \begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$$

ZA PARAMETR s jsem místo 1chy dosadil -1.

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VYPOČÍTEJ SOUŘADNICE VEKTORU

$m = (3; 4; 5)$

POKUD MÁME VEKTORY:

$\vec{u} = (1; 1; 0) \quad \vec{v} = (1; 0; 1) \quad \vec{w} = (0; 1; 1)$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 1 & 0 & 1 & 4 \\ 0 & 1 & 1 & 5 \end{array}\right) \begin{array}{l} \cdot(-1) \\ \swarrow + \\ \\ \end{array} \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & -1 & 1 & 1 \\ 0 & 1 & 1 & 5 \end{array}\right) \begin{array}{l} \\ \cdot 1 \\ \swarrow + \end{array} \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 2 & 6 \end{array}\right) \cdot \tfrac{1}{2} \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 3 \end{array}\right) \begin{array}{l} \\ \nwarrow + \\ \cdot(-1) \end{array} \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & -1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right) \begin{array}{l} \nwarrow + \\ \cdot 1 \\ \\ \end{array} \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right) \cdot(-1) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underline{\underline{\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}}}$$