$$f(x) = e^x \sqrt[3]{x^2} = e^x \cdot x^{\frac{2}{3}}$$

1

$Df = R$

$P_x = ?$ $y = 0$

$$0 = e^x \cdot x^{\frac{2}{3}} \quad / \cdot \frac{1}{e^x}$$

$$\frac{1}{e^x} \cdot 0 = \frac{e^x}{e^x} \cdot x^{\frac{2}{3}}$$

$$0 = 1 \cdot x^{\frac{2}{3}}$$

$$0 = x^{\frac{2}{3}} \quad / \cdot x^{\frac{1}{3}}$$

$$x^{\frac{1}{3}} \cdot 0 = x^{\frac{2}{3}+\frac{1}{3}}$$

$$0 = x^{\frac{3}{3}}$$

$$0 = x$$

$\underline{P_x [0,0]}$

$P_y = ?$ $x = 0$

$$y = e^0 \cdot 0^{\frac{2}{3}}$$

$$y = 1 \cdot 0$$

$$y = 0$$

$\underline{P_y [0,0]}$

② SPOJITOST ? ANO

PERIODICITA ? NE

SUDÁ NEBO LICHÁ ?

$$f(-x) = e^{-x} \cdot (-x)^{\frac{2}{3}}$$

NENÍ SUDÁ, NEVYPADÁ JAKO ZADÁNÍ

$$-\left(e^{-x} \cdot x^{\frac{2}{3}} \right)$$

NENÍ LICHÁ, NEVYPADÁ JAKO ZADÁNÍ

NENÍ SUDÁ ANI LICHÁ

(3) LIMITY V KRAJNÍCH BODECH

$$\overset{\infty \cdot \infty}{\lim_{x \to \infty} \left(e^x \sqrt[3]{x^2} \right)} = +\infty$$

$$\overset{-\infty \cdot \infty}{\lim_{x \to -\infty} \left(e^x \sqrt[3]{x^2} \right)} = -\infty$$

4.1 $f'(x)$

EXTRÉMY

$$f'(x) = \left(e^x \cdot x^{\frac{2}{3}}\right)' =$$

$$= \left(e^x\right)' \cdot x^{\frac{2}{3}} + e^x \cdot \left(x^{\frac{2}{3}}\right)' =$$

$$= e^x \cdot (x)' \cdot x^{\frac{2}{3}} + e^x \cdot \frac{2}{3} x^{-\frac{1}{3}} \cdot (x)' =$$

$$= e^x \cdot x^{\frac{2}{3}} + e^x \cdot \frac{2}{3} x^{-\frac{1}{3}} =$$

$$= e^x \left(x^{\frac{2}{3}} + \frac{2}{3} x^{-\frac{1}{3}}\right)$$

$$x^{\frac{2}{3}} + \frac{2}{3} x^{-\frac{1}{3}} = 0$$

$$\frac{x^{\frac{2}{3}}}{1} + \frac{2}{3\sqrt[3]{x}} = 0$$

$$\frac{3\sqrt[3]{x}\sqrt[3]{x^2} + 2}{3\sqrt[3]{x}} = 0 \quad / \cdot 3\sqrt[3]{x}$$

$$3\sqrt[3]{x}\sqrt[3]{x^2} + 2 = 0$$

$$3\sqrt[3]{x \cdot x^2} + 2 = 0$$

$$3x + 2 = 0 \quad /-2$$

$$3x = -2 \quad /:3$$

$$\underline{x = -\frac{2}{3}}$$

(4.2)

$-\frac{2}{3}$ $\quad$ 0

$e\left(\frac{3x+2}{3\sqrt[3]{x}}\right)$	$(-\infty, -\frac{2}{3})$	$(-\frac{2}{3}, 0)$	$(0, \infty)$
	+	−	+
	↗	↘	↗

LOKÁLNÍ MAXIMUM $\qquad$ LOKÁLNÍ MINIMUM

$$f(x) = e^{x} \cdot x^{\frac{2}{3}}$$

$$f\left(-\frac{2}{3}\right) = e^{-\frac{2}{3}} \cdot \left(-\frac{2}{3}\right)^{\frac{2}{3}} = e^{-\frac{2}{3}} \cdot \sqrt[3]{\left(-\frac{2}{3}\right)^{2}} =$$

$$= \frac{1}{e^{\frac{2}{3}}} \cdot \sqrt[3]{\left(-\frac{2}{3}\right)^{2}} =$$

$$= \frac{1}{2{,}71^{\frac{2}{3}}} \cdot \sqrt[3]{0{,}45} =$$

$$= \frac{1}{1{,}94} \cdot 0{,}77 \doteq$$

MAX $\left[-\frac{2}{3}; 0{,}39\right]$ $\qquad$ $\doteq 0{,}3969$

(VŠUDE ZAOKROUHLOVAL NEBO SI VYBRAL **přibližné** ČÍSLO PRO POČÍTÁNÍ)

$$f(0) = e^{0} \cdot 0^{\frac{2}{3}} = 1 \cdot 0 = 0$$

MIN $[0{,}0]$

5 $f''(x)$ KONVEXNOST KONKÁVNOST

$$f''(x) = \left[e^x\left(x^{\frac{2}{3}} + \frac{2}{3}x^{-\frac{1}{3}}\right)\right]' =$$

$$= (e^x)' \cdot \left(x^{\frac{2}{3}} + \frac{2}{3}x^{-\frac{1}{3}}\right) + e^x \cdot \left(x^{\frac{2}{3}} + \frac{2}{3}x^{-\frac{1}{3}}\right)' =$$

$$= e^x \cdot \left(x^{\frac{2}{3}} + \frac{2}{3}x^{-\frac{1}{3}}\right) + e^x \cdot \left(\frac{2}{3}x^{-\frac{1}{3}} - \frac{2}{9}x^{-\frac{4}{3}}\right) =$$

$$= e^x\left(x^{\frac{2}{3}} + \frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}x^{-\frac{1}{3}} - \frac{2}{9}x^{-\frac{4}{3}}\right) =$$

$$= e^x\left(x^{\frac{2}{3}} + \frac{4}{3}x^{-\frac{1}{3}} - \frac{2}{9}x^{-\frac{4}{3}}\right)$$

$$e^x = 0 \quad / \cdot x$$

$$x e^x = 0 \quad / \cdot \frac{1}{e^x}$$

$$\underline{x = 0}$$

Poznámka:

$$\sqrt[3]{x^4} = \sqrt[3]{x} \cdot \sqrt[3]{x} \cdot \sqrt[3]{x} \cdot \sqrt[3]{x} =$$
$$= \sqrt[3]{x}\,\sqrt[3]{x \cdot x \cdot x} =$$
$$= \sqrt[3]{x}\,\sqrt[3]{x^3} =$$
$$= \sqrt[3]{x} \cdot x$$

$$\frac{\sqrt[3]{x^2}}{1} + \frac{4}{3\sqrt[3]{x}} - \frac{2}{9\sqrt[3]{x^4}} = 0$$

$$\frac{\sqrt[3]{x^2} \cdot 9x\sqrt[3]{x} + 12x - 2}{9x\sqrt[3]{x}} = 0 \quad / \cdot 9x\sqrt[3]{x}$$

$$9x\sqrt[3]{x^3} + 12x - 2 = 0$$

$$9x^2 + 12x - 2 = 0$$

$$9x^2+12x-2=0$$

$$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x_{1,2}=\frac{-12\pm\sqrt{12^2-4\cdot 9\cdot(-2)}}{2\cdot 9}$$

$$x_{1,2}=\frac{-12\pm\sqrt{216}}{18}$$

$\underline{x_1=0{,}15}$

$\underline{x_2=-1{,}5}$

	$(-\infty,-1{,}5)$	$(-1{,}5\,;0)$	$(0;\,0{,}15)$	$(0{,}15\,;+\infty)$
$e^x\,\frac{9x^2+12x-2}{9x\sqrt[3]{x}}$	$+$	$-$	$-$	$+$
	$\cup$	$\cap$	$\cap$	$\cup$

INFLEXNÍ BOD $-1{,}5$

INFLEXNÍ BOD $0{,}15$

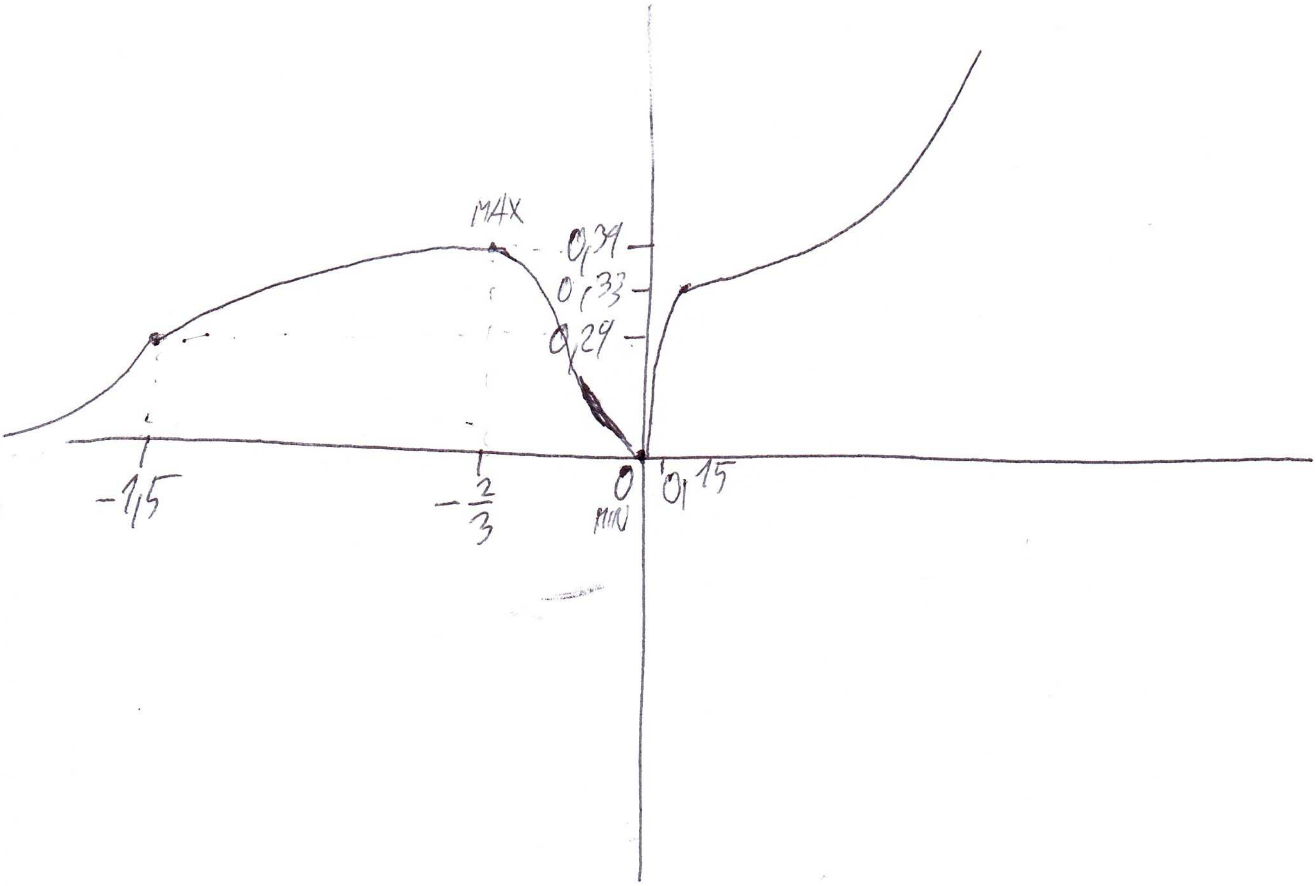
$$f(-1{,}5)=e^{-1{,}5}\cdot(-1{,}5)^{\frac{2}{3}}=$$
$$=e^{-\frac{3}{2}}\cdot\sqrt[3]{(-1{,}5)^2}=$$
$$=\frac{1}{e^{\frac{3}{2}}}\cdot 1{,}31=$$
$$=0{,}22\cdot 1{,}31=0{,}29$$

$[-1{,}5;\,0{,}29]$

$$f(0{,}15)=e^x\cdot x^{\frac{2}{3}}\Rightarrow e^{0{,}15}\cdot 0{,}15^{\frac{2}{3}}=$$
$$=2{,}71^{0{,}15}\cdot 0{,}15^{\frac{2}{3}}=$$
$$=1{,}16\cdot 0{,}28=0{,}33$$

$[0{,}15;\,0{,}33]$

GRAF



ASYMPTOTA: $y = x$

$H_f = (0, +\infty)$