DERIVACE

DERIVACE – JE TO ZMĚNA FUNKČNÍ HODNOTY NĚJAKÉ FCE NA NĚJAKÉM INTERVALU

– JE TO NĚJAKÁ LIMITA

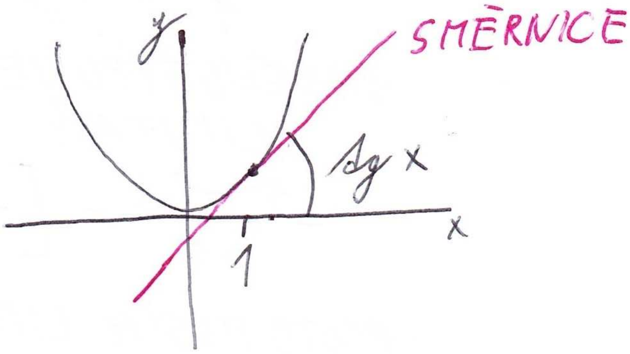
$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \underbrace{f'(a)}_{}$$

DOSTANU DERIVACI FCE V BODĚ a.

ZADÁNÍ

$$\boxed{f(x) = x^2 \text{ "v 1čce" } a = 1}$$

BUDEME SE BLÍŽIT JEDNIČCE



PARABOLA

JAKÁ JE FUNKČNÍ HODNOTA V NULE?

$f(0) = 0$

SMĚRNICE = ? SMĚRNICOVÝ TVAR PŘÍMKY, KDE TADY NAJDU SMĚRNICI?

$y = kx + q$

↑ SMĚRNICE

CO NÁM UDÁVÁ TOTO k VZHLEDEM K PŘÍMCE?

JESTLI BUDE ROSTOUCÍ NEBO KLESAJÍCÍ

JAK BUDE VYPADAT PŘÍMKA: $y = x$?

$y = 2x$?

OVLIVŇUJE k I NÁKLON PŘÍMKY

= UDÁVÁ NÁM (tg) TANGENS ÚHLU KTERÝ TA TEČNA SVÍRÁ S KLADNOU ČÁSTÍ OSY X,

PROTO NÁM k UDÁVÁ NÁKLON

TEDY:

HLEDÁME TANGENS TOHOTO ÚHLU, KTERÝ SVÍRÁ TA TEČNA S TOU OSOU X.

DERIVACE FCE V BODĚ a:

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} =$$

VIZ ZADÁNÍ: FUNKČNÍ HODNOTA V a JE 1.

$$= \lim_{x \to a} \frac{x^2 - 1}{x - 1} =$$

K ČEMU JDE x?

x JDE k a

a JE PRO NÁS CO?

1

JAKÉHO TYPU JE TATO LIMITA?

$$\frac{1^2 - 1}{1 - 1} = \left[\frac{0}{0}\right]$$

PROTO MUSÍM LIMITU UPRAVIT

$$= \left[\frac{0}{0}\right] = \lim_{x \to a} \frac{(x-1)(x+1)}{(x-1)} =$$

POKUD POČÍTÁM LIMITU VE VLASTNÍM BODĚ, VYTÝKAT x NEMÁ SMYSL

$$= \lim_{x \to a} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}} = \lim_{x \to 1} x+1 = 2$$

DERIVACE KVADRATICKÉ FCE V 1ce JE 2.

DERIVACE x^2 JE $2x$ A KDYŽ DOSADÍM 1 ZA X, TAK JE TO 2, COŽ SEDÍ S VÝPOČTEM POMOCÍ LIMITY.

DERIVACE FUNKCE x^2 V JEDNIČCE JE 2.

PŘÍKLAD: (BEZ POČÍTÁNÍ DERIVACE V BODĚ)
ODVOĎ VZOREC PRO DERIVACI FUNKCE $\frac{1}{x}$ V LIBOVOLNÉM BODĚ a.

$$f(x) = \frac{1}{x} \quad a \in R$$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} =$$

$$= \lim_{x \to a} \frac{\frac{1}{x} - \frac{1}{a}}{x - a} =$$

$$= \lim_{x \to a} \frac{\frac{a - x}{xa}}{x - a} = \lim_{x \to a} \frac{a - x}{xa} \cdot \frac{1}{x - a} =$$

$$= \lim_{x \to a} \frac{a - x}{x^2 a - a^2 x} = \lim_{x \to a} \frac{a - x}{xa(x - a)} =$$

$$= \lim_{x \to a} \frac{-x + a}{xa(-1)(-x + a)} = \lim_{x \to a} \frac{\cancel{(-x + a)}}{xa(-1)\cancel{(-x + a)}} =$$

$$= \lim_{x \to a} \frac{1}{-xa} = \lim_{x \to a} -\frac{1}{xa} = -\frac{1}{aa} = -\frac{1}{a^2}$$

$$\boxed{(c)' = 0}$$ DERIVACE KONSTANTNÍ FCE

c je konstanta

$(2)' = 0 \quad (\sqrt{2})' = 0 \quad (\pi)' = 0$

$$\boxed{(x^n)' = n\,x^{n-1} \cdot (x)'}$$

n je konstanta

$$\boxed{(\sin x)' = \cos x \cdot (x)'}$$

$$\boxed{(\cos x)' = -\sin x \cdot (x)'}$$

$$\boxed{(\mathrm{tg}\, x)' = \frac{1}{\cos^2 x} \cdot (x)'}$$

MŮŽE POMOCI PRO ZAPAMATOVÁNÍ

$\mathrm{tg}\, x = \frac{\sin x}{\cos x}$

$$\boxed{(\mathrm{cotg}\, x)' = \frac{-1}{\sin^2 x} \cdot (x)'}$$

$\mathrm{cotg}\, x = \frac{\cos x}{\sin x}$

$$\boxed{(e^x)' = e^x \cdot (x)'}$$

$(e^{2x})' = e^{2x} \cdot (2x)' =$

$= \underline{\underline{2e^{2x}}}$

$$\boxed{(a^x)' = a^x \ln a \cdot (x)'} \qquad (2^x)' = 2^x \ln 2 \cdot (x)'$$

$$\boxed{(\ln x)' = \frac{1}{x} \cdot (x)'} \qquad x \in (0, +\infty)$$

$$\boxed{(\log_a x)' = \frac{1}{x \ln a} \cdot (x)'}$$

$$\boxed{(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \cdot (x)'}$$

$$\boxed{(\arccos x)' = \frac{-1}{\sqrt{1-x^2}} \cdot (x)'}$$

$$\boxed{(\operatorname{arctg} x)' = \frac{1}{1+x^2} \cdot (x)'}$$

$$\boxed{(\operatorname{arccotg} x)' = \frac{-1}{1+x^2} \cdot (x)'}$$

POZNÁMKA: $\ln 1 = 0$

$\ln e = 1$

$$(f \pm g)' = f' \pm g'$$

$$(c \cdot f)' = c \cdot f'$$ c - constanta

$$(f \cdot g)' = f' \cdot g + f \cdot (g)'$$

$$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$$

POZNÁMKA:

$$\left(\frac{s-1}{\sqrt{2}}\right)' =$$

constanta!

$$= (s-1)' \cdot \frac{1}{\sqrt{2}} + (s-1) \cdot \left(\frac{1}{\sqrt{2}}\right)' =$$

$$= 1 \cdot \frac{1}{\sqrt{2}} + (s-1) \cdot 0 =$$

$$= \frac{1}{\sqrt{2}}$$

POKUD JE VE JMENOVATELI CONSTANTA, LZE POUZE TÍMTO ZPŮSOBEM

$$\left(\frac{s-1}{x\sqrt{2}}\right)' =$$ LZE POUŽÍT:

$$(f \cdot g)'$$

$$\left(\frac{f}{g}\right)'$$

$$[f(g(x))]' = f'(g(x)) \cdot g'(x)$$

$$f(x) = 2 - \frac{4}{3}x^3 - \frac{3}{4}x^4$$

$$f'(x) = \underline{\underline{-4x^2 - 3x^3}}$$

VYSVĚTLENÍ:

$$\cancel{2} - 3\,\frac{4}{3}\,x^{3-1} - 4\,\frac{3}{4}\,x^{4-1}$$

CONSTANTA

VYNÁSOBÍM

VŽDY ODEČTU -1

$$f(x) = 5^x + \log_3 x$$

$$f'(x) = \underline{\underline{5^x \cdot \ln 5 + \frac{1}{x \ln 3}}}$$

$$f(x) = \frac{1}{x} + \frac{2}{x^2} + \frac{3}{x^3}$$

$$f(x) = x^{-1} + 2x^{-2} + 3x^{-3}$$

$$f'(x) = -1x^{-2} + (-4)x^{-3} + (-9)x^{-4} =$$

$$= \underline{\underline{-x^{-2} - 4x^{-3} - 9x^{-4}}}$$

$$f(x) = x + \sqrt{x} + \sqrt[3]{x}$$

$$f(x) = x + x^{\frac{1}{2}} + x^{\frac{1}{3}}$$

$$f'(x) = \underline{\underline{1 + \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{3}x^{-\frac{2}{3}}}}$$

$$\boxed{f(x) = \frac{2x - \sqrt[3]{x} + 3}{\sqrt{x}}}$$

$$f(x) = \frac{2x - x^{\frac{1}{3}} + 3}{1} \cdot \frac{1}{x^{\frac{1}{2}}} = \left(2x - x^{\frac{1}{3}} + 3\right) \cdot x^{-\frac{1}{2}} =$$

$$= 2x^{\frac{1}{2}} - x^{-\frac{1}{6}} + 3x^{-\frac{1}{2}}$$

$$f'(x) = x^{-\frac{1}{2}} - \left(-\frac{1}{6}\right) x^{-\frac{7}{6}} + \left(-\frac{3}{2}\right) x^{-\frac{3}{2}}$$

$$f'(x) = \underline{\underline{x^{-\frac{1}{2}} + \frac{1}{6} x^{-\frac{7}{6}} - \frac{3}{2} x^{-\frac{3}{2}}}}$$

$$\left(\frac{1}{\sqrt{x}} + \frac{1}{6} \frac{1}{\sqrt[6]{x^7}} - \frac{3}{2} \frac{1}{\sqrt{x^3}}\right)$$

$$\boxed{f(x) = x^2 \cdot \sin x}$$

$$f'(x) = (x^2)' \cdot \sin x + x^2 \cdot (\sin x)'$$

$$f'(x) = \underline{\underline{2x \sin x + x^2 \cos x}}$$

$$\boxed{f(x) = \ln x \cdot \operatorname{arctg} x}$$

$$f'(x) = (\ln x)' \cdot \operatorname{arctg} x + \ln x \cdot (\operatorname{arctg} x)'$$

$$f'(x) = \frac{1}{x} \cdot (x)' \cdot \operatorname{arctg} x + \ln x \cdot \frac{1}{1+x^2}$$

$$f'(x) = \underline{\underline{\frac{1}{x} \operatorname{arctg} x + \ln x \cdot \frac{1}{1+x^2}}}$$

$$\boxed{f(x)=\frac{\cos x}{x^2+1}}$$

$$f'(x)=\frac{(\cos x)'(x^2+1)-\cos x\cdot(x^2+1)'}{(x^2+1)^2}=$$

$$=\underline{\underline{\frac{-\sin x\,(x^2+1)-\cos x\,(2x)}{(x^2+1)^2}}}$$

$$\boxed{f(x)=\frac{\sin x-x\cos x}{\cos x+x\sin x}}$$

$$f'(x)=\frac{(\sin x-x\cos x)'\cdot(\cos x+x\sin x)-(\sin x-x\cos x)\cdot(\cos x+x\sin x)'}{(\cos x+x\sin x)^2}=$$

$$=\frac{[(\sin x)'-(x\cos x)']\cdot(\cos x+x\sin x)-(\sin x-x\cos x)\cdot[(\cos x)'+(x\sin x)']}{(\cos x+x\sin x)^2}=$$

$$=\frac{\{\cos x-[(x)'\cdot\cos x+x\cdot(\cos x)']\}(\cos x+x\sin x)-(\sin x-x\cos x)\cdot\{-\sin x+[(x)'\sin x+x\cdot(\sin x)']\}}{(\cos x+x\sin x)^2}=$$

$$=\frac{\{\cos x-[1\cdot\cos x+x\cdot(-\sin x)]\}(\cos x+x\sin x)-(\sin x-x\cos x)\cdot\{-\sin x+[1\cdot\sin x+x\cos x]\}}{(\cos x+x\sin x)^2}=$$

$$=\frac{(\cos x-\cos x+x\sin x)(\cos x+x\sin x)-(\sin x-x\cos x)(-\sin x+\sin x+x\cos x)}{(\cos x+x\sin x)^2}=$$

$$=\frac{(x\sin x)(\cos x+x\sin x)-(\sin x-x\cos x)(x\cos x)}{(\cos x+x\sin x)^2}=$$

$$= \frac{[x\sin(x)\cos(x) + x^2\sin^2 x] - [x\sin(x)\cos(x) - x^2\cos^2 x]}{(\cos x + x\sin x)^2} =$$

$$= \frac{x\sin(x)\cos(x) + x^2\sin^2 x - x\sin(x)\cos(x) + x^2\cos^2 x}{(\cos x + x\sin x)^2} =$$

$$= \frac{x^2\sin^2 x + x^2\cos^2 x}{(\cos x + x\sin x)^2} =$$

$$= \frac{x^2(\sin^2 x + \cos^2 x)}{(\cos x + x\sin x)^2} =$$

$$= \frac{x^2 \cdot 1}{(\cos x + x\sin x)^2} = \frac{x^2}{(\cos x + x\sin x)^2}$$

$$\boxed{\left(e^{-x}\cdot \operatorname{arctg}\sqrt{x}\right)'} =$$

$$= \left(e^{-x}\right)'\cdot \operatorname{arctg}\sqrt{x} + e^{-x}\cdot\left(\operatorname{arctg}\sqrt{x}\right)' =$$

$$= e^{-x}\cdot(-x)'\cdot \operatorname{arctg}\sqrt{x} + e^{-x}\cdot \frac{1}{1+(\sqrt{x})^2}\cdot(\sqrt{x})' =$$

$$= e^{-x}\cdot(-1)\cdot \operatorname{arctg}\sqrt{x} + e^{-x}\cdot\frac{1}{1+x}\cdot\left(x^{\frac{1}{2}}\right)' =$$

$$= -e^{-x}\cdot \operatorname{arctg}\sqrt{x} + \frac{e^{-x}}{1+x}\cdot\frac{1}{2}\cdot x^{-\frac{1}{2}} =$$

$$= -e^{-x}\cdot \operatorname{arctg}\sqrt{x} + \frac{e^{-x}}{2(1+x)}\cdot\frac{1}{\sqrt{x}} =$$

$$= -e^{-x}\cdot \operatorname{arctg}\sqrt{x} + \frac{e^{-x}}{2\sqrt{x}(1+x)} =$$

$$= \frac{e^{-x}}{2\sqrt{x}(1+x)} - e^{-x}\cdot \operatorname{arctg}(\sqrt{x})$$

$$Df = \langle 0, +\infty)$$

$$Df' = (0, +\infty)$$

$$\boxed{f(x) = \frac{x^{\frac{7}{6}} \cdot \operatorname{cotg} x}{\ln x}}$$

$$f'(x) = \frac{\left(x^{\frac{7}{6}} \cdot \operatorname{cotg} x\right)' \cdot \ln x - \left(x^{\frac{7}{6}} \cdot \operatorname{cotg} x\right) \cdot (\ln x)'}{(\ln x)^2} =$$

$$= \frac{\left[\left(x^{\frac{7}{6}}\right)' \cdot \operatorname{cotg} x + x^{\frac{7}{6}} \cdot (\operatorname{cotg} x)'\right] \cdot \ln x - \left(x^{\frac{7}{6}} \cdot \operatorname{cotg} x\right) \cdot \frac{1}{x} \cdot (x)'}{(\ln x)^2} =$$

$$= \frac{\left(\frac{7}{6} x^{\frac{1}{6}} \operatorname{cotg} x + x^{\frac{7}{6}} \frac{-1}{\sin^2 x}\right) \cdot \ln x - \left(x^{\frac{7}{6}} \cdot \operatorname{cotg} x\right) \cdot \frac{1}{x}}{(\ln x)^2}$$

DERIVACE SLOŽENÉ FCE:

$$\boxed{f(x) = (2x-4)^7}$$

VNĚJŠÍ $z = y^7$

VNITŘNÍ $y = 2x - 4$

$$z' = (y^7)' \cdot y' = 7y^6 \cdot y' = 7y^6 \cdot (2x-4)' = 7y^6 \cdot 2 =$$
$$= 7(2x-4)^6 \cdot 2 =$$
$$= 14(2x-4)^6$$

$$\left((2x-4)^7\right)' = 7(2x-4)^6 \cdot (2x-4)' =$$
$$= 7(2x-4)^6 \cdot 2$$
$$= \underline{\underline{14(2x-4)^6}}$$

$$\boxed{f(x) = \ln(3x+4)}$$

VNĚJŠÍ $z = \ln y$
VNITŘNÍ $y = 3x+4$

$$z' = (\ln y)' \cdot y' = \frac{1}{y} \cdot 3 = \frac{1}{3x+4} \cdot 3$$

NEBO:

$$(\ln(3x+4))' = \frac{1}{3x+4} \cdot (3x+4)' = \underline{\underline{\frac{1}{3x+4} \cdot 3}}$$

$$\boxed{f(x) = \sqrt{\sin x}} = (\sin x)^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}(\sin x)^{-\frac{1}{2}} \cdot (\sin x)' =$$

$$= \underline{\underline{\frac{1}{2} \sin x^{-\frac{1}{2}} \cdot \cos x}}$$

$$\boxed{f(x) = e^{-x^2}}$$

VNĚJŠÍ $z = e^y$
VNITŘNÍ $y = -x^2$

$$f'(x) = e^{-x^2} \cdot (-x^2)' =$$

$$= \underline{\underline{e^{-x^2} \cdot (-2x)}}$$

$$\boxed{f(x) = 5^{4x}} \quad 5^{4x} \cdot \ln 5 \cdot (4x)' = \underline{\underline{5^{4x} \cdot \ln 5 \cdot 4}}$$

$$\boxed{f(x) = \log^3 x^2} = (\log x^2)^3$$

$$f'(x) = 3(\log x^2)^2 \cdot (\log x^2)' =$$

$$= 3(\log x^2)^2 \cdot \frac{1}{x^2 \ln 10} \cdot (x^2)' =$$

$$= \underline{\underline{3(\log x^2)^2 \cdot \frac{1}{x^2 \cdot \ln 10} \cdot 2x}}$$

$$\boxed{f(x) = \cot g\,(2x)}$$

$$f'(x) = \frac{-1}{\sin^2 2x} \cdot (2x)' =$$

$$= -\frac{1}{\sin^2 2x} \cdot 2 =$$

$$= \underline{\underline{-\frac{2}{\sin^2(2x)}}}$$

$$\boxed{f(x) = \sin(2x)}$$

$$f'(x) = \cos 2x \cdot (2x)'$$

$$= \underline{\underline{2\cos(2x)}}$$

$$\boxed{f(x) = \ln \operatorname{tg} \frac{x}{2}}$$

$$f'(x) = \frac{1}{\operatorname{tg}\frac{x}{2}} \cdot \left(\operatorname{tg}\frac{x}{2}\right)' =$$

$$= \frac{1}{\operatorname{tg}\frac{x}{2}} \cdot \frac{1}{\cos^2\frac{x}{2}} \cdot \left(\frac{x}{2}\right)' =$$

$$= \frac{1}{\operatorname{tg}\frac{x}{2}} \cdot \frac{1}{\cos^2\frac{x}{2}} \cdot \left(x \cdot \frac{1}{2}\right)' =$$

$$= \frac{1}{\operatorname{tg}\frac{x}{2}} \cdot \frac{1}{\cos^2\frac{x}{2}} \cdot \left[(x)' \cdot \frac{1}{2} + x \cdot \left(\frac{1}{2}\right)'\right] =$$

$$= \underline{\underline{\frac{1}{\operatorname{tg}\frac{x}{2}} \cdot \frac{1}{\cos^2\frac{x}{2}} \cdot \frac{1}{2}}}$$

$$\boxed{f(x) = 2^{\operatorname{cotg}\frac{1}{x}}}$$

$$f'(x) = 2^{\operatorname{cotg}\frac{1}{x}} \ln 2 \cdot \left(\operatorname{cotg}\frac{1}{x}\right)' =$$

$$= 2^{\operatorname{cotg}\frac{1}{x}} \ln 2 \cdot \frac{-1}{\sin^2\left(\frac{1}{x}\right)} \cdot \left(\frac{1}{x}\right)' =$$

$$= 2^{\operatorname{cotg}\frac{1}{x}} \ln 2 \cdot \frac{-1}{\sin^2\left(\frac{1}{x}\right)} \cdot \left[\frac{(1)' \cdot x - 1 \cdot (x)'}{x^2}\right] =$$

$$= 2^{\operatorname{cotg}\frac{1}{x}} \ln 2 \cdot \frac{-1}{\sin^2\left(\frac{1}{x}\right)} \cdot \left(-\frac{1}{x^2}\right) =$$

$$= 2^{\operatorname{cotg}\frac{1}{x}} \ln 2 \cdot \frac{-1}{\sin^2\left(\frac{1}{x}\right)} \cdot (-1) \cdot \frac{1}{x^2} =$$

$$= 2^{\operatorname{cotg}\frac{1}{x}} \ln 2 \cdot \frac{-1}{\sin^2\left(\frac{1}{x}\right)} \cdot (-1) \cdot x^{-2} =$$

$$= \underline{\underline{2^{\operatorname{cotg}\frac{1}{x}} \ln 2 \cdot \left(-\frac{1}{\sin^2\left(\frac{1}{x}\right)}\right) \cdot \left(-x^{-2}\right)}}$$

$$\boxed{f(x) = \ln\left(\frac{1}{\sqrt{\sin e^{x^2+3}}}\right)}$$

$$f'(x) = \frac{1}{\frac{1}{\sqrt{\sin e^{x^2+3}}}} \cdot \left(\frac{1}{\sqrt{\sin e^{x^2+3}}}\right)' =$$

$$= \frac{1}{1} \cdot \frac{\sqrt{\sin e^{x^2+3}}}{1} \cdot \left(\frac{1}{(\sin e^{x^2+3})^{\frac{1}{2}}}\right)' =$$

$$= \sqrt{\sin e^{x^2+3}} \cdot \left[(\sin e^{x^2+3})^{-\frac{1}{2}}\right]' =$$

$$= \sqrt{\sin e^{x^2+3}} \cdot \left(-\frac{1}{2}\right)(\sin e^{x^2+3})^{-\frac{3}{2}} \cdot (\sin e^{x^2+3})' =$$

$$= \sqrt{\sin e^{x^2+3}} \cdot \left(-\frac{1}{2}\right)(\sin e^{x^2+3})^{-\frac{3}{2}} \cdot \cos e^{x^2+3} \cdot (e^{x^2+3})' =$$

$$= \sqrt{\sin e^{x^2+3}} \cdot \left(-\frac{1}{2}\right)(\sin e^{x^2+3})^{-\frac{3}{2}} \cdot \cos e^{x^2+3} \cdot e^{x^2+3} \cdot (x^2+3)' =$$

$$= \sqrt{\sin e^{x^2+3}} \cdot \left(-\frac{1}{2}\right)(\sin e^{x^2+3})^{-\frac{3}{2}} \cdot \cos e^{x^2+3} \cdot e^{x^2+3} \cdot 2x$$

$$\boxed{f(x) = x^x}$$

TADY SE MĚNÍ ZÁKLAD I MOCNINA, PROTO NELZE POUŽÍT VZORCE:

$$(a^x)' = a^x \ln a \cdot (x)'$$

$$(x^n)' = n\, x^{n-1}$$

PODLE VZORCŮ DERIVOVAT NELZE, PROTO JI MUSÍM UPRAVIT.

① CELOU ROVNICI ZLOGARITMUJI

$$y = x^x$$

$$\ln y = \ln x^x$$

② UPRAVÍM

$$\underbrace{\ln y} = x \ln x$$

③ POUŽIJI INVERZNÍ FCI.

$$\boxed{\ln y = \log_e y}$$

$$\underbrace{\log_e y} = x \ln x$$

④ POTREBUJI Z TOHO DOSTAT ZPĚT y

$$\boxed{\begin{aligned} \log_a b &= c \\ b &= a^c \end{aligned}}$$

$$y = e^{x \ln x}$$

$$\begin{aligned} y' &= e^{x\ln x} \cdot (x \ln x)' = \\ &= e^{x \ln x} \cdot [(x)' \cdot \ln x + x \cdot (\ln x)'] = \\ &= e^{x \ln x} \cdot [\ln x + x \cdot \tfrac{1}{x} \cdot (x)'] = \\ &= e^{x \ln x} \cdot (\ln x + 1) = \\ &= \underline{\underline{x^x (\ln x + 1)}} \end{aligned}$$

$$\boxed{f(x)=\sqrt{x+\sqrt{x-\sqrt{x}}}}$$

$$f'(x)=\left(\left\{x+\left[x-(x)^{\frac{1}{2}}\right]^{\frac{1}{2}}\right\}^{\frac{1}{2}}\right)'=$$

$$=\frac{1}{2}\left\{x+\left[x-(x)^{\frac{1}{2}}\right]^{\frac{1}{2}}\right\}^{-\frac{1}{2}}\cdot\left\{x+\left[x-(x)^{\frac{1}{2}}\right]^{\frac{1}{2}}\right\}'=$$

$$=\frac{1}{2}\left\{x+\left[x-(x)^{\frac{1}{2}}\right]^{\frac{1}{2}}\right\}^{-\frac{1}{2}}\cdot\left\{(x)'+\left\{\left[x-(x)^{\frac{1}{2}}\right]^{\frac{1}{2}}\right\}'\right\}=$$

$$=\frac{1}{2}\left\{x+\left[x-(x)^{\frac{1}{2}}\right]^{\frac{1}{2}}\right\}^{-\frac{1}{2}}\cdot\left\{1+\frac{1}{2}\left[x-(x)^{\frac{1}{2}}\right]^{-\frac{1}{2}}\cdot\left[x-(x)^{\frac{1}{2}}\right]'\right\}=$$

$$=\frac{1}{2}\left\{x+\left[x-(x)^{\frac{1}{2}}\right]^{\frac{1}{2}}\right\}^{-\frac{1}{2}}\cdot\left\{1+\frac{1}{2}\left[x-(x)^{\frac{1}{2}}\right]^{-\frac{1}{2}}\cdot\left[1-\frac{1}{2}x^{-\frac{1}{2}}\right]\right\}=$$

$$=\frac{1}{2}\left[x+\left(x-x^{\frac{1}{2}}\right)^{\frac{1}{2}}\right]^{-\frac{1}{2}}\cdot\left[1+\frac{1}{2}\left(x-x^{\frac{1}{2}}\right)^{-\frac{1}{2}}\cdot\left(1-\frac{1}{2}x^{-\frac{1}{2}}\right)\right]$$

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ZKRÁCENĚ: $\boxed{\left(f(x)^{g(x)}\right)' = e^{g(x)\ln f(x)} \cdot \left(g(x)\ln f(x)\right)'}$

VZOREC TAM KDE SE MĚNÍ ZÁKLAD I MOCNINA

KDE LZE POUŽÍT

například: $(x)^x$

$(2x)^{3x}$

POZNÁMKA:

$\boxed{(2x)^{3x}}$ $\left(= 2^{3x} \cdot x^{3x} = 8^x \cdot x^{3x}\right)$

$$y' = e^{3x\ln 2x} \cdot (3x\ln 2x)' =$$

$$= e^{3x\ln 2x} \cdot \left[(3x)' \cdot \ln 2x + 3x \cdot (\ln 2x)'\right] =$$

$$= e^{3x\ln 2x} \cdot \left[3\ln 2x + 3x \cdot \frac{1}{2x} \cdot (2x)'\right] =$$

$$= e^{3x\ln(2x)} \cdot \left[3\ln(2x) + 3\right]$$

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