BINOMICKÁ VĚTA

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$$\boxed{(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k} \qquad \boxed{\binom{n}{k} = \frac{n!}{(n-k)!\,k!}}$$

ZADÁNÍ: $(1+i)^7$

$$(1+i)^7 = \binom{7}{0} 1^{7-0} i^0 + \binom{7}{1} 1^6 i^1 + \binom{7}{2} 1^5 i^2 + \binom{7}{3} 1^4 i^3 + \binom{7}{4} 1^3 i^4 + \binom{7}{5} 1^2 i^5 + \binom{7}{6} 1^1 i^6 +$$

$$+ \binom{7}{7} 1^0 i^7 =$$

VÍME:

$$\binom{7}{0} = \binom{7}{7} = 1$$

$$\binom{7}{1} = \binom{7}{6} = 7$$

$$\binom{7}{2} = \binom{7}{5} = 21$$

$$\binom{7}{3} = \binom{7}{4} = 35$$

UKÁZKA:

$$\binom{7}{0} = \frac{7!}{(7-0)!\,0!} = \frac{7!}{7!\,1} = \frac{7!}{7!} = \frac{1}{1} = 1$$

$$\binom{7}{1} = \frac{7!}{(7-1)!\,1!} = \frac{7!}{6!\,1} = \frac{7!}{6!} = \frac{7 \cdot 6!}{6!} = \frac{7}{1} = 7$$

$$\binom{7}{5} = \frac{7!}{(7-5)!\,5!} = \frac{7!}{2!\,5!} = \frac{7 \cdot 6 \cdot 5!}{2!\;5!} = \frac{7 \cdot 6}{2!} = \frac{7 \cdot 6}{2 \cdot 1!} = \frac{42}{2} = 21$$

$$\binom{7}{4} = \frac{7!}{(7-4)!\,4!} = \frac{7 \cdot 6 \cdot 5 \cdot 4!}{3!\;4!} = \frac{7 \cdot 6 \cdot 5}{3!} = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1!} = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} = \frac{7 \cdot 6 \cdot 5}{6} = 35$$

VÍME: $\boxed{\begin{array}{l} i^1 = i \\ i^2 = -1 \end{array}}$

$i^3 = i \cdot i^2 = i \cdot (-1) = -i$

$i^4 = (i^2)^2 = (-1)^2 = 1$

$i^5 = i \cdot (i^2)^2 = i \cdot (-1)^2 = i \cdot 1 = i$

$i^6 = (i^2)^3 = (-1)^3 = -1$

$i^7 = i \cdot (i^2)^3 = i\,(-1)^3 = i \cdot (-1) = -i$

$$= 1 \cdot 1 \cdot 1 + 7 \cdot 1 \cdot i + 21 \cdot 1 \cdot i^2 + 35 \cdot 1 \cdot i^3 + 35 \cdot 1 \cdot i^4 + 21 \cdot 1 \cdot i^5 + 7 \cdot 1 \cdot i^6 + 1 \cdot 1 \cdot i^7 =$$

$$= 1 + 7i - 21 - 35i + 35 + 21i - 7 - i =$$

$$= \underline{\underline{-8i + 8}}$$